



[Predicting] the number of gravitational arcs in clusters:  
redshift evolution and scaling of the cross section

**J. Estrada**

*Fermi National Accelerator Laboratory*



DARK ENERGY  
SURVEY



Fermilab

**E. Roulet, S. Mollerach**

*Instituto Balseiro - Centro Atómico Bariloche/CNEA*



Instituto  
Balseiro  
Bariloche

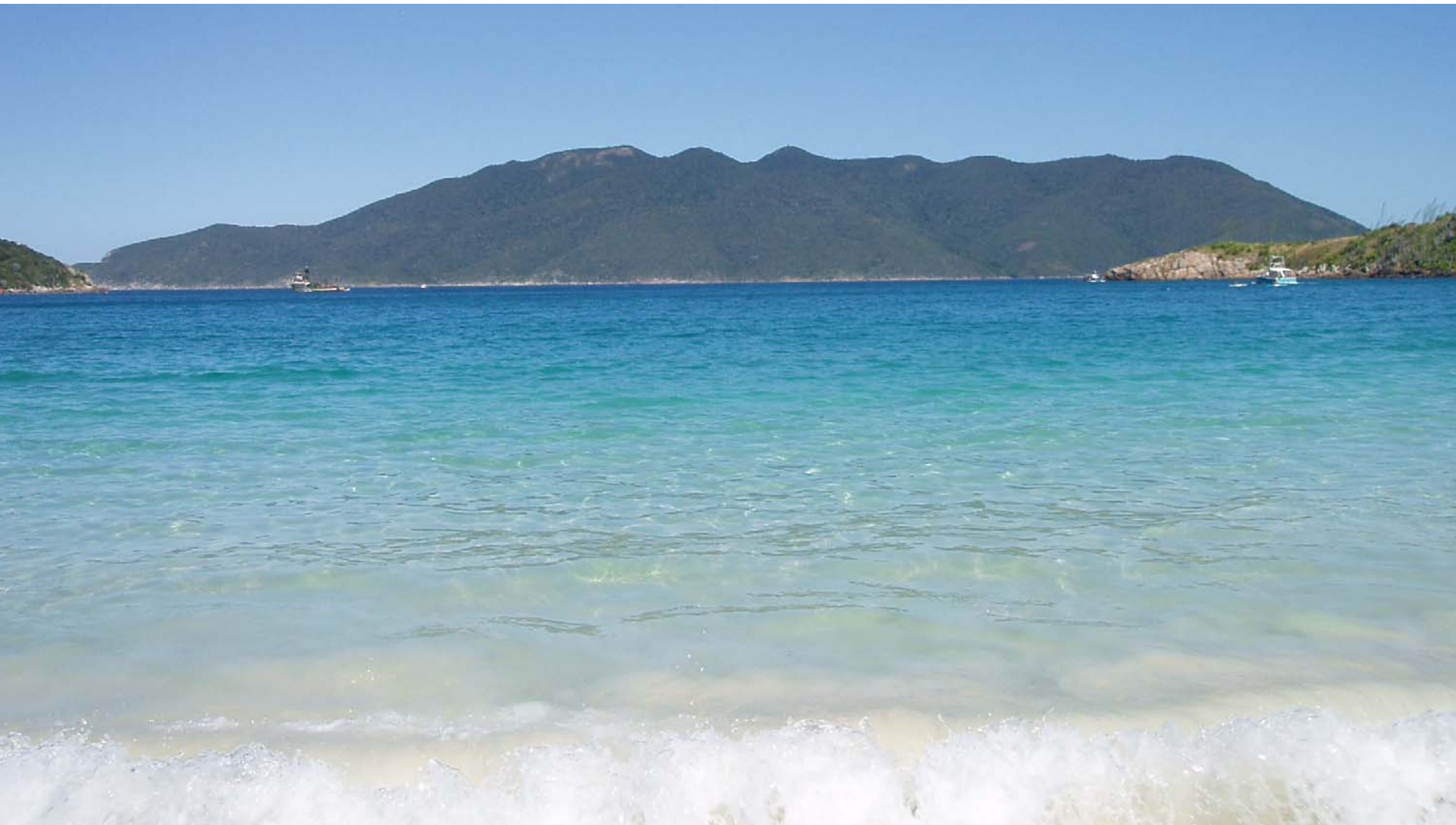




Martín Makler

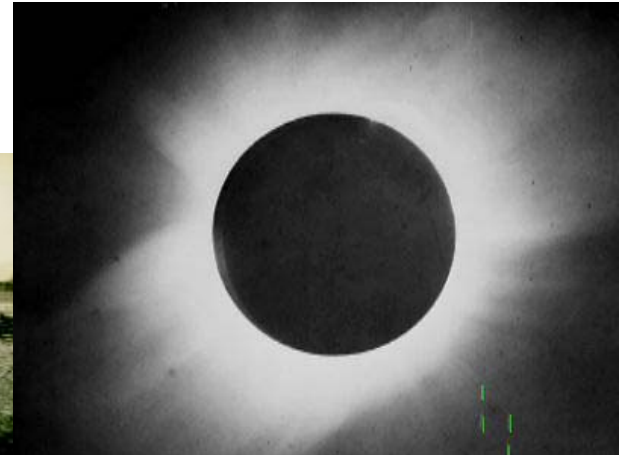
*Centro Brasileiro de Pesquisas Físicas*

Ministério da  
Ciência e Tecnologia



# Motivation: Brief History of the Detection of the Light Deflection by Gravity

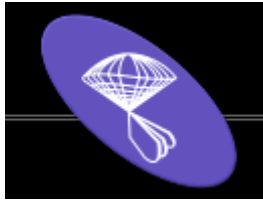
- 1912: Argentinean expedition to observe solar eclipse in Brazil
  - Got rained out
- 1914: Erwin Freundlich's expedition to observe an eclipse in Crimea (Russia)
  - Detained because WWI break up
- 1919: Sobral (Brazil)
  - Detection of the bending of light (better weather than Prince Island)



“The question that my mind formulated was answered by the sunny sky of Brazil”

*A. Einstein*

# Motivation



- Systematic arc search in SDSS clusters/imaging:
  - Vic Scarpine's poster (Estrada et al. 2007)
  - Efficiency of visual inspection and automated code
  - No arcs found!
- Apparent excess at high  $z$ 
  - Dalal, Holder, Hennawi, et al.
  - EMSS, RCS, RCS2
- All the interesting physics you all know about
  - Lens structure
  - Cosmology
  - Etc.
- Upcoming wide field (deeper, better seeing) surveys
  - Dark Energy Survey



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# Observable: Giant Arc Abundance

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- Average number of arcs per cluster  
(e.g. Horesh, Oguri)
- Why?
  - Relative distribution (Hennawi)
  - Does not depend on the mass function  
("WMAP independent")
  - **Does not depend on the cluster selection method** (if not correlated with arcs...)
  - Does not need source redshift
- Goal: address  $z$  evolution with simple model

**"All arc statistics computations are  
either wrong or incomplete"**

**Rocky Kolb**

# Expected number of arcs **per cluster**

- Number of arcs per cluster (pictorial)

$$N_{\text{arcos}} = \int \sigma(M, \dots; z_S, z_L; R, \mu) N(z_S; m_{\text{lim}}) f(R, m) dz_S d\mu dR$$

cross section  
for arc formation

source surface  
density

Selection function  
(detection efficiency)



# Cross section in “Simulations”

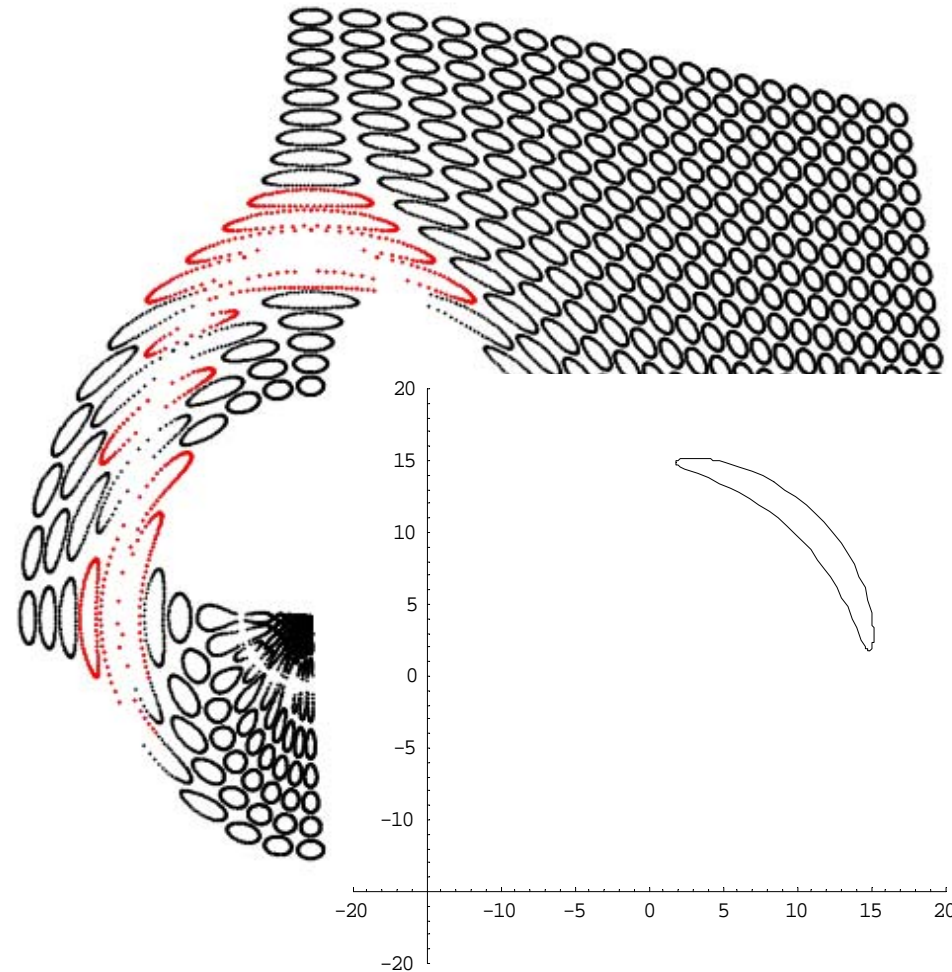
- NFW density profile

$$\rho = \frac{\rho_s}{\left(r / r_s\right)\left(1 + r / r_s\right)^2}$$

- Elliptical density distribution

$$r^2 = x^2 + \frac{y^2}{(1 - e)^2}$$

- sources: ellipses
- gravlens modeling
- Image measurement
  - Algorithm based on analytic “arc function”



- Application of the cuts



# “Local” cross section

- Eigenvalues of mapping

$$\mu_1 = \frac{1}{1 - \kappa + \gamma}, \mu_2 = \frac{1}{1 - \kappa - \gamma}$$

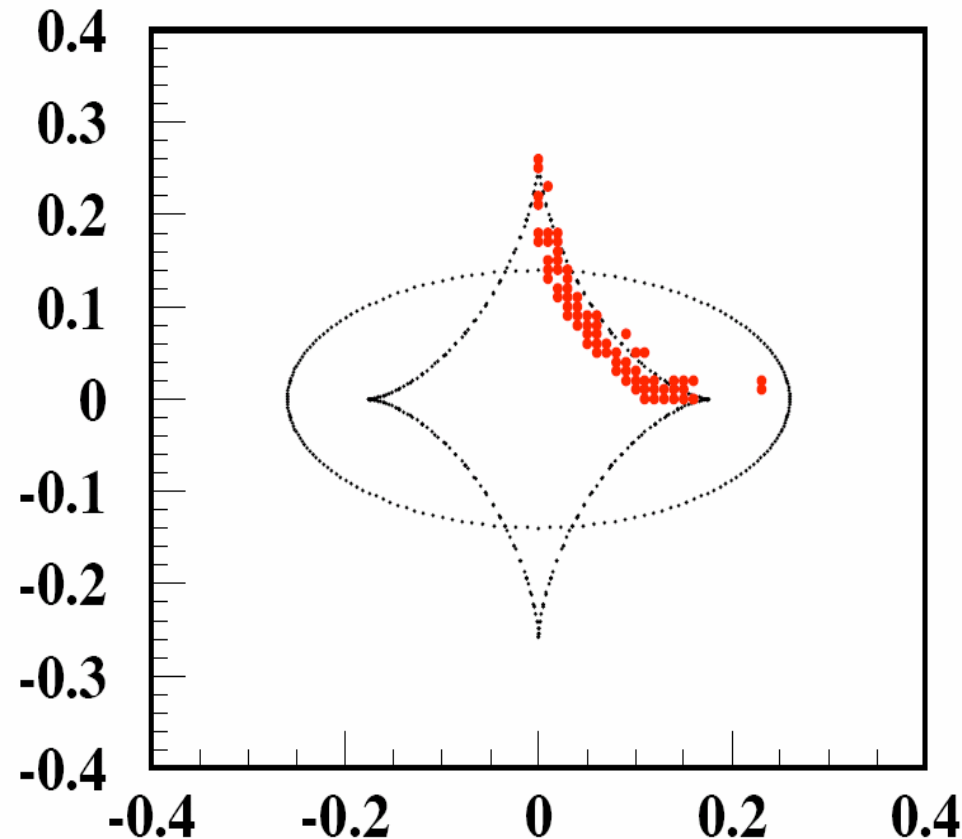
- Magnification and axial ratio

$$\mu = \mu_1 \mu_2; \quad R = \frac{L}{W} = \max \left( \left| \frac{\mu_1}{\mu_2} \right|, \left| \frac{\mu_2}{\mu_1} \right| \right)$$

- ➡ Close to caustics

- Only tangential arcs

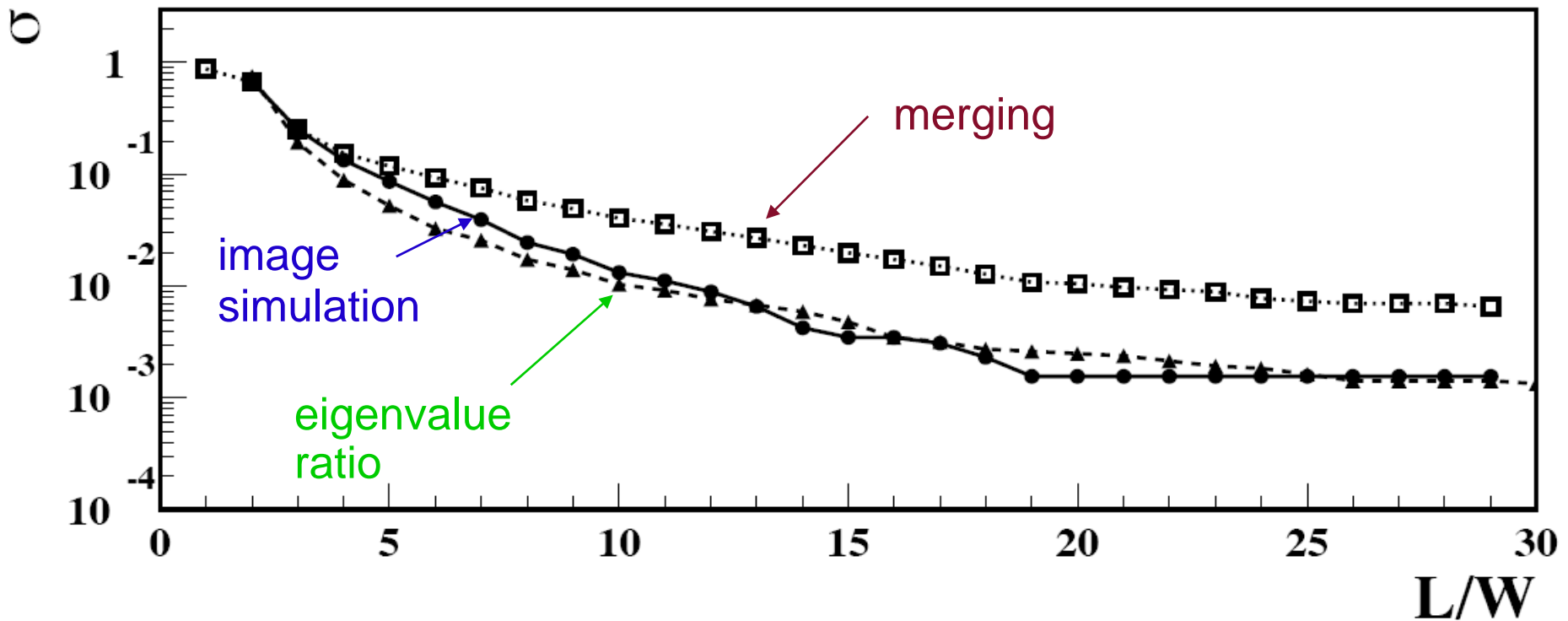
$$\sigma = \int_{\Omega(R > R_{th})} d^2\theta \frac{1}{\mu(\theta)}$$



- Much faster!

# Local x Finite

- ➡ Eigenvalue ratio: good when there is no merging



- To simplify: proceed with eigenvalue ratio

# NFW Cross Section

- Scaling with  $\kappa_s$  and  $r_s$  (Oguri et al. 2003):

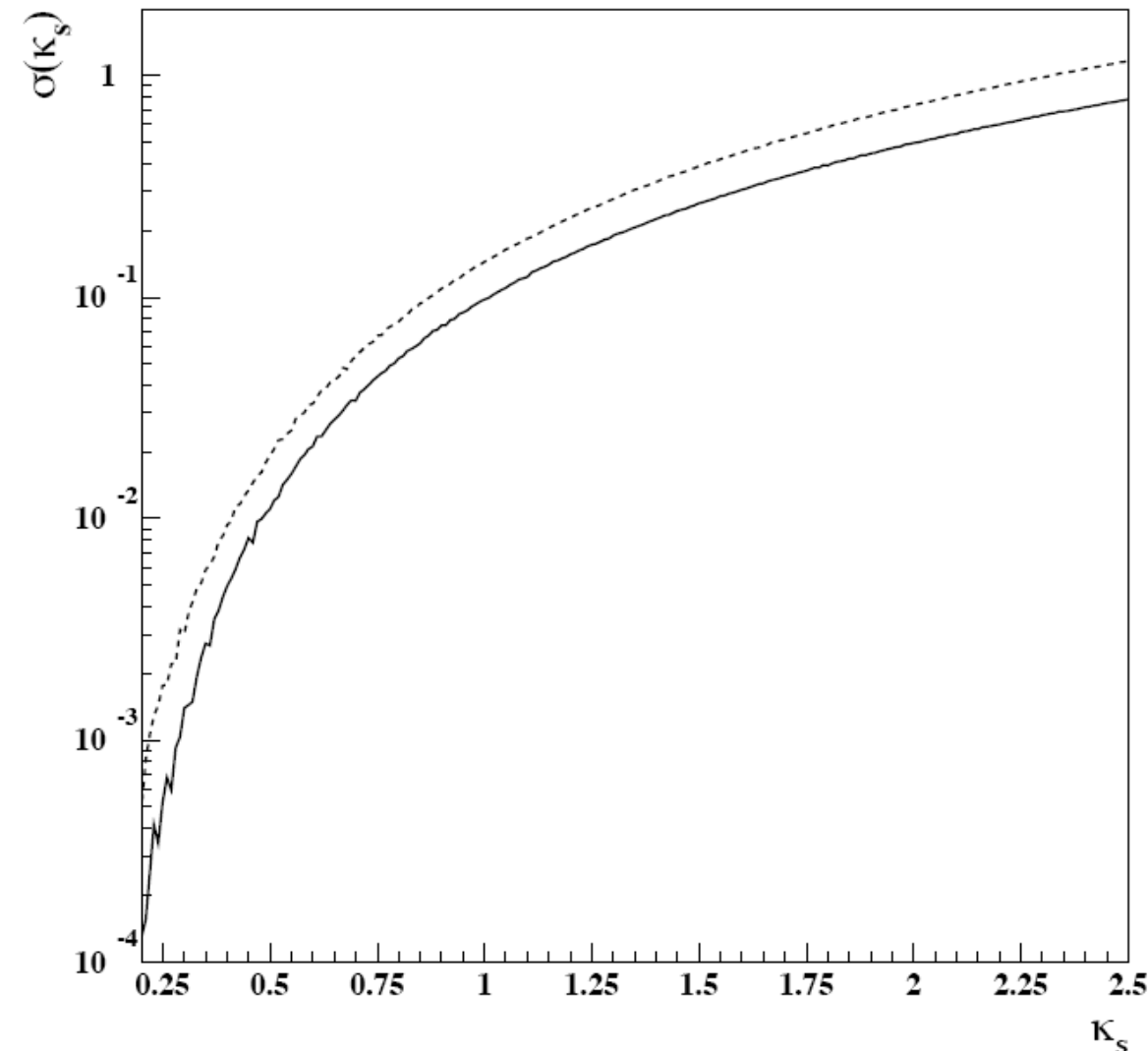
$$\sigma(M, c, e; z_L, z_S; R_{th}, \mu_{lim}) = \tilde{\sigma}(k_s, e; R_{th}, \mu_{lim}) \left( \frac{r_s}{D_L} \right)^2$$

$$\kappa_s = 7.36 \times 10^{-4} \frac{c^2}{\ln(1+c) - c/(1+c)} (\Delta_{vir} \Omega_{M0})^{2/3}$$

$$(1+z)^2 E(z)^{2/3} \left( \frac{M_{vir}}{10^{14} M_{\odot}} h \right)^{1/3} \frac{I_{LS} I_{OL}}{I_{OS}},$$

Analogously  
for  $r_s$

# NFW Cross Section



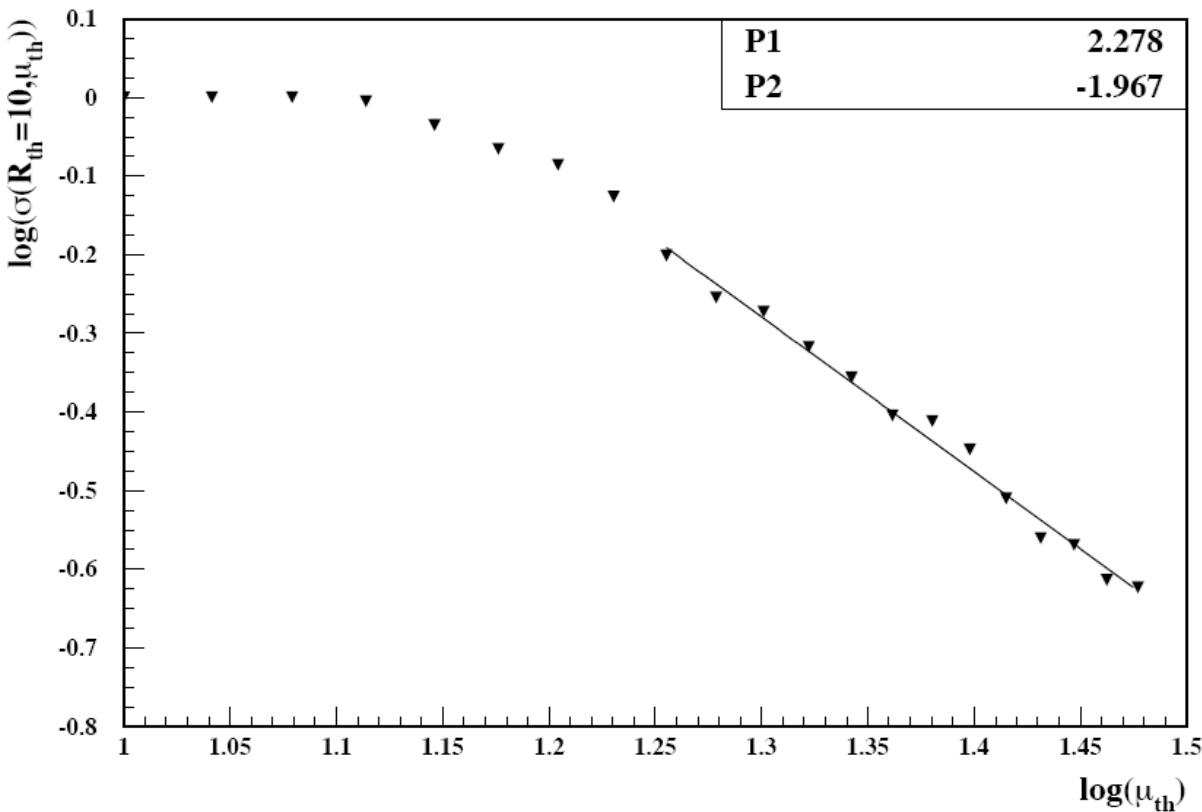
Nonlinear function of  $\kappa_s$  and hence of  $D_{\text{eff}}$

Threshold effect

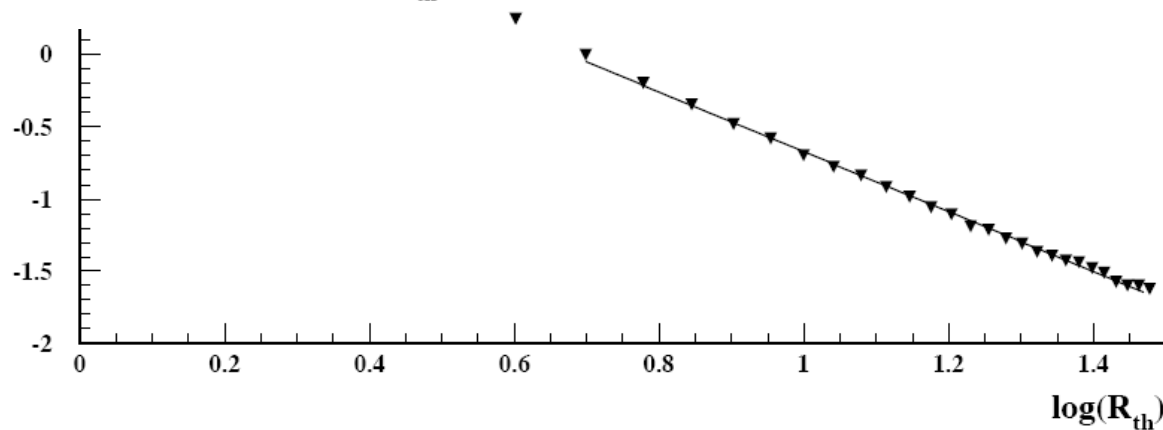
# Scaling of the Cross Section

- Scaling with magnification **is** important:
  - grav. lensing conserves surface brightness
  - but there is noise!  $S/N \propto \sqrt{A} \propto \sqrt{\mu}$
- Scaling with  $L/W$  **is** important:
  - seeing:  $L' = \sqrt{L^2 + \theta_s^2}$ ,  $W' = \sqrt{W^2 + \theta_s^2}$
- Analytical results:
  - Close to folds:  $\mu_T \propto \frac{1}{\sqrt{\beta - \beta_c}}$
  - Thus  $\sigma \propto \mu^{-2}$ ,  $\sigma \propto R^{-2}$

# Scaling of the Cross Section



For fixed L/W ratio



For any  $\mu$

Searching For Strong Lenses

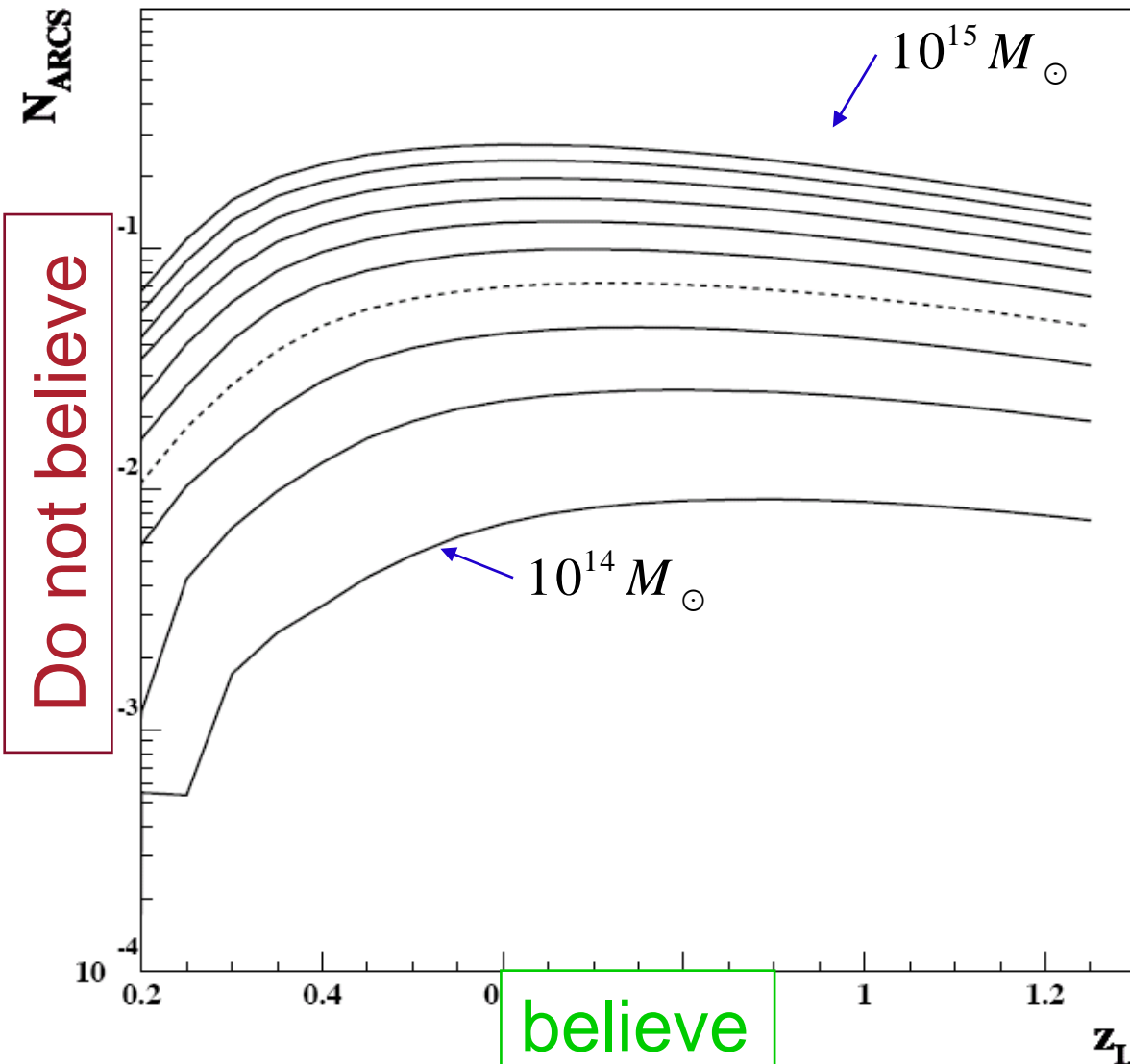
# Result

$$\frac{d\sigma(M, e; z_L, z_S; R_{th}, \mu_{th})}{d\mu_{th}} = \tilde{\sigma}(\kappa_s, e) \left( \frac{r_s}{D_L} \right)^2 \left( 2 \frac{\mu_{\min}^2}{\mu_{th}^3} \right)$$

$$N_{arcs} = \int \int \frac{d\sigma}{d\mu_{th}} n(z_S, m_{\lim} + 2.5 \log \sqrt{\mu_{th}}) dz_S d\mu_{th}$$



# Result



- Large variation with  $z$ 
  - Threshold effect with  $M$ ,  $e$
- Connection to observed discrepancy?

# Concluding Remarks

- Takes into account:
  - $N(z)$ , detection efficiency, magnification
- Conjecture:
  - For single plane lensing
  - ⇒  $\exists$  “Universal cross section”:

$$\sigma(M, c, e \dots; z_L, z_S; R_{th}, \mu_{lim}) \propto \tilde{\sigma}(k_s, e, \dots) \left( \frac{r_s}{D_L} \right)^2 \left( \frac{\mu_{min}^2}{\mu_{lim}^2} \right) \left( \frac{1}{R_{th}^2} \right)$$

- Seen in numerical simulations (Rozo)
  - Can be generalized if cusps are important
- Use/test scalings with  $M$ ,  $z_L$ ,  $z_S$ ,  $\mu$ ,  $R$ , etc. with simulated clusters